INSTITUT NATIONAL DE RECHERCHE EN INFORMATIQUE ET EN AUTOMATIQUE

Preliminary models for Moving Server Networks

Larissa G. AFANASSIEVA - Guy FAYOLLE
Leonid V. NAZAROV

N° 2469
Janvier 1995

PROGRAMME 1

Rapport de recherche

Les rapports de recherche de l'INRIA sont disponibles en format postscript sous ftp.inria.fr (192.93.2.54)

si vous n'avez pas d'accès ftp la forme papier peut être commandée par mail :
e-mail : dif.gesdif@inria.fr
(n'oubliez pas de mentionner votre adresse postale).

par courrier :
Centre de Diffusion
INRIA
BP 105 - 78153 Le Chesnay Cedex (FRANCE)

INRIA research reports are available in postscript format ftp.inria.fr (192.93.2.54)

if you haven't access by ftp we recommend ordering them by e-mail :
e-mail : dif.gesdif@inria.fr
(don't forget to mention your postal address).

by mail :
Centre de Diffusion
INRIA
BP 105 - 78153 Le Chesnay Cedex (FRANCE)

Modèles Préliminaires pour des Réseaux à Serveurs Mobiles

Larissa G. Afanassieva * Guy Fayolle † Leonid V. Nazarov ‡

Janvier 1995

Résumé

On présente trois modèles probabilistes (simples par ordre de complexité croissante) pour des réseaux avec serveurs *mobiles*. On utilise la théorie classique des files d'attente et des méthodes d'analyse asymptotique. Les applications concernent directement des réseaux de véhicules en libre service, type PRAXITÈLE.

*Moscow State University, Mechanico-Mathematical Faculty, Chair of probability, Vorobyevy Gori, 119899 Moscow, RUSSIA.

†INRIA - Domaine de Voluceau, Rocquencourt - BP.105 - 78153 Le Chesnay Cedex - FRANCE.

‡Moscow State University, Faculty of Cybernetics and Numerical Mathematics, Vorobyevy Gori, 119899 Moscow, RUSSIA.

Preliminary models for Moving Server Networks

Larissa G. Afanassieva [*] Guy Fayolle [†] L. V. Nazarov [‡]

Jannuary 1995

Abstract

Three simple stochastic models are proposed for moving server networks, by increasing order of complexity. The mathematical tools rely on classical queueing theory and asymptotic analysis. These models are applied to service vehicle systems (like PRAXITELE).

[*] Moscow State University, Mechanico-Mathematical Faculty, Chair of probability, Vorobyevy Gori, 119899 Moscow, RUSSIA.

[†] INRIA - Domaine de Voluceau, Rocquencourt - BP.105 - 78153 Le Chesnay Cedex - FRANCE.

[‡] Moscow State University, Faculty of Cybernetics and Numerical Mathematics, Vorobyevy Gori, 119899 Moscow, RUSSIA.

1 Introduction

Transportation problems have been for a long time a source of interesting mathematical problems. New transportation problems now arise in the context of demand driven systems (e.g flexible route bus systems, flexible delivery, rental and self service cars, etc.). In these systems, the demand cannot be modelled by simple deterministic methods and stochastic models seem suitable to carry out analysis of performance. This paper is only a preliminary attempt to provide some rough understanding in the behaviour of so-called *service vehicle networks* (SVN). It can be viewed as a creation of models. Some of them are classical and rely on so-called product form networks; others can hardly be solved analytically, but asymptotics when the size of the system increases can be obtained.

Basically, a customer arrives at a station (*node, station, parking lot*), where he takes a car, if any available one, to reach some other station (destination), where he leaves the car. Section 2 is devoted to the simplest case, when the number of available cars is unbounded. Here time dependent parameters are allowed. The model of section 3 assumes a finite number V of cars, no capacity constraints and no waiting room for customers, who are lost if there is no car when they arrive. The last section 4 presents the more interesting (and difficult) case: at each station, the capacity for parking places is limited and there is also a finite waiting room for clients at each station. An asymptotic analysis is presented, in which the number of cars and the numder of nodes simultaneously increase.

2 Non time-homogeneous input process and infinite-server queues

Consider an open queueing network with N stations. At time t, customers arrive at node $i, i = 1, \ldots, N$ according to a non time-homogeneous Poisson process, with deterministic parameter $(\lambda_i(t), t \geq 0)$. These N processes are supposed to be independent. A customer arriving at node i is provided with a car and goes to node j with some probability p_{ij}, so that the matrix $P = (p_{ij})$ be stochastic. In this model, the number of available cars is supposed to be unlimited. After having reached his destination, a customer leaves the network.

Let us introduce the following random variables, for all $i, j = 1, \dots, N$:

- τ_{ij}, the time to go from node i to node j, the corresponding distribution function being $B_{ij}(x)$;
- $x_{ij}(t)$, the number of cars which, at time t, are on their way from i to j.

Now to each link (origin-destination pair) (i, j) with $p_{ij} > 0$, we associate a fictitious node having an infinite number of servers, the service-time of each server being equal to τ_{ij}.

As an immediate corollary of standard results, which likely can be traced to Palm [2], we have the following:

Theorem 2.1 *Assume that all service times, routing and arrival processes are are mutually independent and that $\lambda_i(t), t \geq 0, i, j = \overline{1, N}$ are integrable functions. Then, for each t, $x_{ij}(t), i, j = \overline{1, N}$ are independent random variables having a Poisson distribution, with finite mean*

$$m_{ij}(t) = Ex_{ij}(t) = E \int_{t-\tau_{ij}}^{t} p_{ij}\lambda_i(s)ds.$$

Remark 1 *These results are valid under more general assumptions, including time-dependent routing matrix. The reader is referred to [3] for an extensite survey an some new developments on these problems.*

Example 1 Let $B_{ij}(x) = B(x) = P\{\tau < x\}$ for all $i, j = \overline{1, N}$. Then the total number of moving cars at time t is

$$M(t) = E \int_{t-\tau}^{t} \lambda(s)ds,$$

where $\lambda(s) = \sum_{i=1}^{N} \lambda_i(s)$. The extreme values of $M(t)$ occur at all time instants t_0, satisfying the equation

$$\lambda(t_0) = \int_0^{t_0} \lambda(t_0 - u)dB(u).$$

3 Finite number of servers and no waiting room

3.1 Model description

Here we consider a more realistic situation, where the total number of cars in the system is finite, say V. Customers who do not find available cars at the station where they have arrived are lost. Moreover, a car arriving at a parking lot waits until the arrival of the next customer (no *empty returns*); then both leave to reach their destination. Here we shall assume time-independent arrival rates $\lambda_i, i, j = \overline{1, N}$. All other parameters and notations in this network are the same as above.
As in the first model, we introduce virtual nodes denoted by (i, j). A car will be said to *be*

- at node (i, j), if he is moving from station i to station j;
- at station i, if it is waiting for a customer in parking lot i.

Then the system can be viewed as a *closed* network with $N(N+1)$ nodes, in which V cars are moving around and there are two kinds of nodes:

1. the nodes of type i, which are single-server queues with exponentially distributed service-times, with parameter $\lambda_i, i = \overline{1, N}$;
2. the nodes denoted by pairs (i, j), which are depicted as infinite-sever queues, with service-time distribution function $B_{ij}(x)$.

Let $x_i(t)$, $i = \overline{1, N}$, be the number of cars parked in queue i at time t and $x_{ij}(t)$, $i, j = \overline{1, N}$, - the number of cars moving between i and j.
When $\tilde{\tau}_{ij} = E\tau_{ij} < \infty$ and matrix P $= [p_{ij}]$ is ergodic,its invariant measure being denoted by $\pi = (\pi_1, \ldots, \pi_N)$, it is well known that the vector-process

$$X(t) = (x_i(t), i = \overline{1, N};\ x_{ij}(t), i, j = \overline{1, N})$$

has an equilibrium distribution. Indeed, we have the following

Theorem 3.1 *Under the above assumptions, the stationary distribution of $X(t)$ has a product form given by*

$$\lim_{t \to \infty} P\left(x_i(t) = n_i, i = \overline{1, N}; x_{ij}(t) = n_{ij}, i, j = \overline{1, N}\right) =$$

$$P\left(n_i, i=\overline{1,N}; n_{ij}, i,j=\overline{1,N}\right) = C\prod_{k=1}^{N}\left(\frac{\pi_k}{\lambda_k}\right)^{n_k}\prod_{i=1}^{N}\prod_{j=1}^{N}\frac{(\pi_i p_{ij}\tilde{\tau}_{ij})^{n_{ij}}}{n_{ij}!}, \tag{3.1}$$

where

$$\sum_{i=1}^{N} n_i + \sum_{i=1}^{N}\sum_{j=1}^{N} n_{ij} = V,$$

and C is a normalizing constant.

Proof. The statement of the theorem is an immediate corollary of classical results (see e.g. [4]). It suffices to remark that the transition probabilities through the network can be written as $p^\star_{\alpha,\beta}$, where $\alpha,\beta = i$ or (i,j) and

$$p^\star_{ij} = 0, \quad p^\star_{i,(ij)} = p_{ij}, \quad p^\star_{(ij),j} = 1.$$

Then

$$P(n_i, i=\overline{1,N}; n_{ij}, i,j=\overline{1,N}) = C\prod_{i=1}^{N} g_i(n_i)\prod_{i=1}^{N}\prod_{j=1}^{N} g_{ij}(n_{ij}),$$

where

$$g_i(n_i) = \left(\frac{l_i}{\lambda_i}\right)^{n_i}, \; g_{ij}(n_{ij}) = \frac{(l_{ij}\tilde{\tau}_{ij})^{n_{ij}}}{n_{ij}!},$$

and

$$l_j = \sum_{j=1}^{N} l_{ji} p^\star_{(ji),i} = \sum_{j=1}^{N} l_{ji}, \; l_{ji} = l_j p_{ji}.$$

Thus $l_i = \pi_i$ and $l_{ij} = \pi_i p_{ij}$, $\forall i,j = 1,\ldots,N$

The probability that a customer arriving at queue i be not served (*loss probability*) is exactly

$$P^{(i)}_{loss} = \sum_{\sum_{j\neq i} k_j + \sum_{i,j} k_{ij} = V} P(k_1,\ldots,k_{i-1},0,k_{i+1},\ldots,k_N;\, k_{ij}).$$

3.2 Symmetrical network

When the network is symmetric, we have

$$p_{ij} = \frac{1}{N},\ \lambda_i = \lambda,\ \tau_{ij} = \tau,\ i,j = \overline{1,N}.$$

Let $g(t)$ be the number of moving cars at time t. Now the probability that a car goes to station j is equal to $\frac{1}{N}$, no matter which station the car was coming from: thus it suffices to introduce only one global virtual node, which will be assigned number 0 and contains all moving cars. The transition probabilities are then simply

$$p^\star_{ij} = 0,\ i,j = \overline{1,N};\ p^\star_{i0} = 1;\ p^\star_{0i} = \frac{1}{N},\ i = \overline{1,N};\ p^\star_{00} = 0.$$

The equilibrium state distribution is obtained as in the previous theorem:

$$\lim_{t\to\infty} P\left(x_0(t) = k_0, \ldots, x_N(t) = k_N\right) = P(k_0, \ldots, k_N) \ = \ C\lambda^{-V+k_0} = \frac{(N\tau)^{k_0}}{k_0!},$$

where $\sum_{i=0}^{N} k_i = V$. The constant C is given by

$$C = \lambda^V \left(\sum_{m=0}^{V} C^m_{N-1+m} \frac{\rho^{V-m}}{(V-m)!} \right)^{-1},$$

where $\rho \ = \ N\lambda\tau$. Then the loss probability is given by

$$P_{loss} \ = \ \sum_{k_0+\cdots+k_{N-1}=V} p(k_0, \ldots, k_{N-1}, 0) \ = \ C\lambda^V \sum_{m=0}^{V} C^m_{N-2+m} \frac{\rho^{V-m}}{(V-m)!} =$$

$$\frac{\displaystyle\sum_{m=0}^{V} C^m_{N-2+m} \frac{\rho^{V-m}}{(V-m)!}}{\displaystyle\sum_{m=0}^{V} C^m_{N-1+m} \frac{\rho^{V-m}}{(V-m)!}}. \qquad (3.2)$$

3.3 Asymptotic estimate of the loss probability in the symmetrical case

It is of practical interest to compute P_{loss} in (3.2), when the number of cars increases with the number of stations. We shall take $V = rN$.
Let us introduce

$$u(s,l;x) = \sum_{m=0}^{s} C_{l+m}^{m} \frac{x^{s-m}}{(s-m)!}.$$

Then

$$P_{loss} \stackrel{\text{def}}{=} \varphi(N) = \frac{u(V,N-2;\rho)}{u(V;N-1;\rho)} = \frac{u(rN,N-2;N\lambda\tau)}{u(rN,N-1;N\lambda\tau)}.$$

One can check that $u(s,l;x)$ admits the following integral representation

$$u(s,l;x) = \frac{1}{2i\pi}\int_C \frac{e^{xt}\,dt}{t^s(1-t)^{l+1}}.$$

From this representation, exact asymptotic expansions can be obtained by using the classical *saddle-point* method in the complex plane. Simply quoting the main steps, we can write

$$u(rN,N-2;N\lambda\tau) = \frac{1}{2i\pi}\int_C \frac{e^{N\lambda\tau t}\,dt}{t^{rN}(1-t)^{N-1}}.$$

Analogously,

$$u(rN,N-1;N\lambda\tau) = \frac{1}{2i\pi}\int_C \frac{e^{N\lambda\tau t}\,dt}{t^{rN}(1-t)^{N}}.$$

It follows (skipping all intermediate derivations – see for instance [5]) that

$$\varphi(N) = 1 - \frac{2r}{\lambda\tau + r + 1 + \sqrt{(\lambda\tau + r + 1)^2 - 4\lambda\tau r}} + O(\frac{1}{N}). \qquad (3.3)$$

3.4 Comparison of analytic and simulation results

The results derived in this section for symmetrical networks were compared with simulation experiments carried out at INRIA by F. Dumontet, who used the package QNAP. The parameters had the following values:

$$r = 10,\ \lambda = 10,\ \tau = 1,\ N = \overline{5,50}.$$

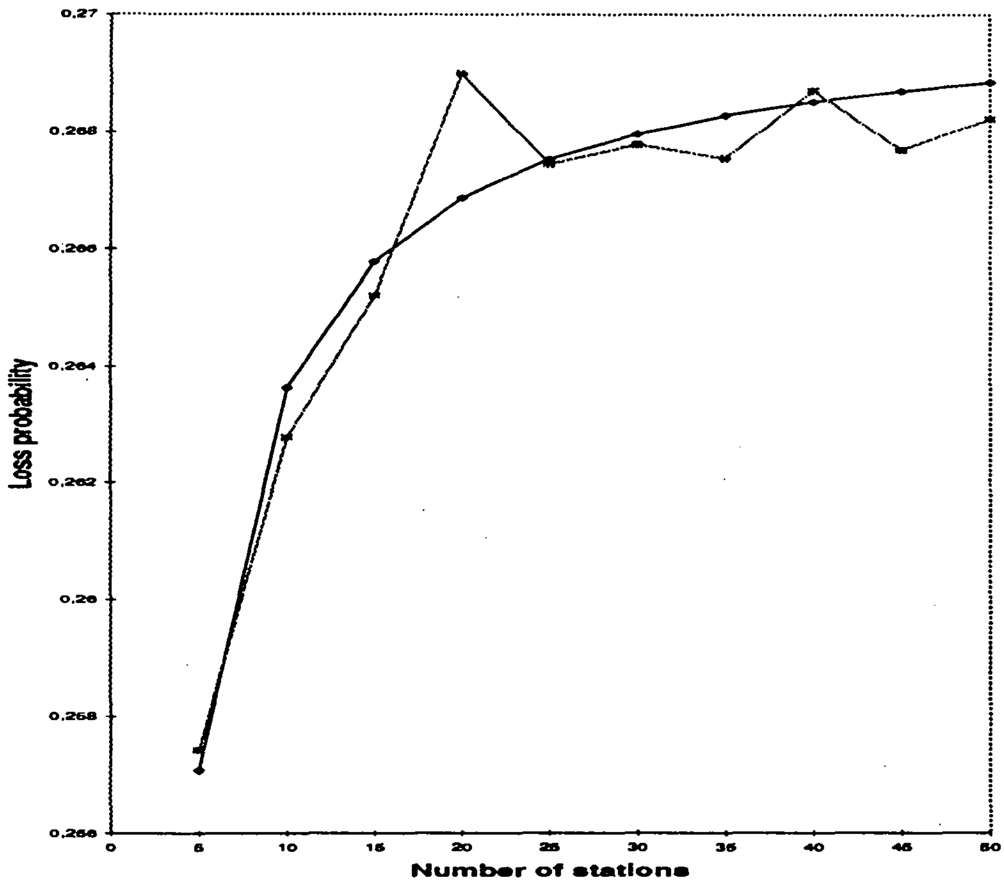

Figure 1: Comparison of analytical and simulation results

On figure 1, the loss probability has been drawn as a function of the number of nodes. Both simulation and analytic results show that this function is monotone increasing.

4 Finite capacities for cars and customers

4.1 Model description

This last model tries to cope with still more realistic assumptions. As a consequence, there is a tremendous jump in terms of analytic complexity and closed-form solutions can hardly be expected. Nonetheless, it is possible to get interesting information on the behaviour of the system when its size becomes large.
There are N parking lots. Arrivals of cars (resp. customers) at the i-th station form a time-homogeneous Poisson process with rate α_i (resp. λ_i), $i = 1, \ldots, N$. Let k_i be the capacity of the i-th parking lot and m_i be the size of the waiting room for customers at station $i-$th node. Matrix $P = \{p_{ij}\}$, as before, denotes the routing probabilities of customers and τ_{ij} is the average time to travel from node i to node j. A client arriving at a given node waits for a car, if there are free waiting places; otherwise he leaves the network and is lost. Similarly, a car arriving at a parking lot with free parking slots stays there (if there are no waiting customers); otherwise, if there is no free space, then the car immediately goes to node j with probability p_{ij}.

The basic characteristics of this network are listed thereafter:

L - average number of cars waiting in parking lots;
L_0 - average number of cars moving without clients;
θ - average time cars are waiting in parking lots;
γ - average time necessary to looking for a free parking place at the end of a trip;
P_{loss} - the probability of loosing customers;
W - average customer- waiting time.

Assumption A Throughout this chapter, we shall assume that, when the number of nodes in the above network tends to infinity, each of them behaves approximately as a birth and death process. No attempt to present a proof of this fact will be made, since it requires a deep study, which will be presented in a forthcoming paper.

4.2 Analysis of a node in isolation

Take one arbitrary node in the network (its number will be omitted for the sake of brevity). Let $x(t)$ (resp. $y(t)$) be the number of cars (resp. customers) in this node at time t. Define the process

$$z(t) = \begin{cases} k - x(t), & \text{if } x(t) > 0,\ y(t) = 0, \\ k + y(t), & \text{if } y(t) > 0,\ x(t) = 0, \\ k, & \text{if } x(t) = 0,\ y(t) = 0. \end{cases} \tag{4.1}$$

From the preceding Assumtion **A**, $z(t)$ is a birth-death process with finite state-space and, for h sufficiently small, we have

$$\begin{array}{ll} P(z(t+h) = i+1 | z(t) = i) = \lambda h + o(h), & 0 \le i < m+k, \\ P(z(t+h) = i-1 | z(t) = i) = \alpha h + o(h), & 0 < i \le m+k, \\ P(z(t+h) = m+k+1 | z(t) = m+k) = 0, & \\ P(z(t+h) = -1 | z(t) = 0) = 0. & \end{array}$$

Let

$$p_n = \lim_{t \to \infty} P(z(t) = n), \tag{4.2}$$

so that

$$p_n = \frac{\rho^n (1-\rho)}{1 - \rho^{k+m+1}}, \quad n = \overline{0, k+m}, \tag{4.3}$$

where have set $\rho = \lambda / \alpha$.

Remark 2 *The following holds:*

1. *For $k = \infty$, a nonzero limit (4.2) exists if, and only if, $\rho < 1$.*
2. *For $m = \infty$, a nonzero limit (4.2) exists if, and only if, $\rho > 1$.*
3. *If both $k = \infty$ and $m = \infty$, then $p_n = 0, \forall n$.*

Return now to the components $x(t)$ and $y(t)$ of the process $z(t)$. By (4.1) and (4.3), we have

$$S_n \stackrel{\text{def}}{=} \lim_{t \to \infty} P(x(t) = n) = p_{k-n} = \frac{\rho^{k-n}(1-\rho)}{1 - \rho^{k+m+1}}, \quad n = \overline{1, k},$$

and we set

$$S_0 \stackrel{\text{def}}{=} \sum_{n=k}^{k+m} p_n = \frac{\rho^k(1-\rho^{m+1})}{1-\rho^{k+m+1}}. \tag{4.4}$$

Similarly,

$$Q_n \stackrel{\text{def}}{=} \lim_{t\to\infty} P(y(t)=n) = p_{k+n} = \frac{\rho^{k+n}(1-\rho)}{1-\rho^{k+m+1}},\ n=\overline{1,m},$$

with

$$Q_0 \stackrel{\text{def}}{=} \sum_{n=0}^{k} p_n = \frac{1-\rho^{k+1}}{1-\rho^{k+m+1}}. \tag{4.5}$$

The following characteristics can be obtained:

- The average number of cars waiting in parking lots:

$$L = \sum_{j=1}^{k} \frac{\rho^{k-j}(1-\rho)}{1-\rho^{k+m+1}} j = \frac{(k+1)(1-\rho)-1+\rho^{k+1}}{(1-\rho)(1-\rho^{k+m+1})}. \tag{4.6}$$

- The average number of waiting customers:

$$M = \frac{\rho^{k+1}(1-(m+1)\rho^m + m\rho^{m+1})}{(1-\rho)(1-\rho^{k+m+1})}. \tag{4.7}$$

- The probability of loosing customers:

$$P_{loss} = Q_m = \frac{\rho^{k+m}(1-\rho)}{1-\rho^{k+m+1}}. \tag{4.8}$$

- The probability of *empty return*:

$$S_k = \frac{1-\rho}{1-\rho^{k+m+1}}. \tag{4.9}$$

Now all basic characteristics of the network can be derived from the relations (4.4) and (4.5), provided that one can compute all intensities α_j ($j=\overline{1,N}$). The next sections show that this is indeed possible.

4.3 Fully symmetrical network

Suppose that

$$p_{ij} = \frac{1}{N},\ \tau_{ij} = \tau, \lambda_i = \lambda \ \text{ and } V = rN,$$

where r is a positive number. Then the average number of moving cars is equal to $rN - NL$, where L has been defined in (4.6), and the intensity of the arrival process of cars at a node (equilibrium equation) is

$$\alpha = (r - L)/\tau, \ \text{ so that } L \ = \ r \ - \ \frac{\lambda\tau}{\rho}. \tag{4.10}$$

It suffices then to substitute the unique non negative root ρ_0 of (4.10) (if any) in (4.6)-(4.9) to obtain all the basic characteristics of the network.

Denote

$$L(\rho) \ = \ \frac{(k+1)(1-\rho) - 1 + \rho^{k+1}}{(1-\rho)(1-\rho^{m+k+1})}.$$

This function is monotone decreasing and

$$L(0) = k,\ L(1) = \lim_{\rho \to 1} L(\rho) = \frac{k(k+1)}{2(k+m+1)},\ L(\infty) = 0.$$

The function $\psi(\rho) = r - \lambda\tau/\rho$ monotonically increases and

$$\psi(0) = -\infty,\ \psi(\rho) > 0,\ if\ \rho > \frac{\lambda\tau}{r},\ \psi(\infty) = r.$$

Consider the following various situations:

1. If $r \leq \lambda\tau$, then $\rho_0 > \frac{\lambda\tau}{r} \geq 1$ and $\alpha_0 = \lambda/\rho_0 < \lambda$.

2. If $\lambda\tau < r < \frac{k(k+1)}{2(k+m+1)} + \lambda\tau$, then again $\rho_0 > 1$.

3. if $r > \frac{k(k+1)}{2(k+m+1)} + \lambda\tau$, then the root of equation (4.10) is less than 1. This means that $\alpha > \lambda$, i.e. cars arrive at a higher rate than customers: they will be frequently free, causing thus a bad utilization of the resources in the network.

It is also now not too difficult to determine network parameters in order to optimize the following (natural) efficiency criterion. Per unit of time, let c_1 (resp. c_2) be the cost of loosing a customer (resp. a car) and c_3, the profit coming from the transportation of a customer. The total equivalent cost per unit of time is

$$W(k,m,r,\lambda\tau) = \lambda c_1 Q_m + c_2 L - c_3\lambda(1-Q_m). \tag{4.11}$$

This function could be optimized in terms of k and r.

4.4 Approximate solutions of equation 4.10

Our intention is to analyze equation (4.10), which reads.

$$\frac{(k+1)(1-\rho)-1+\rho^{k+1}}{(1-\rho)(1-\rho^{m+k+1})} = r - \frac{\lambda\tau}{\rho}.$$

- For $\rho > 1$ and $m \to \infty$, equation (4.10) is equivalent to $r = \lambda\tau/\rho$. So, when m is large, $\rho \sim \lambda\tau/r$.

- Let now $m = 0$ and $k \to \infty$. here there are no capacity constraints for parking lots, but there is no waiting room for customers: this is the case analyzed in section 3.2. Then (4.10) takes the form

$$\frac{1}{\rho-1} = r - \frac{\lambda\tau}{\rho},$$

or, equivalently,

$$r\rho^2 - (\lambda\tau + r + 1)\rho + \lambda\tau = 0, \tag{4.12}$$

$$\rho_0 = \frac{\lambda\tau + r + 1 \pm \sqrt{(\lambda\tau + r + 1)^2 - 4\lambda\tau r}}{2r}.$$

It is not difficult to check that one of the roots of equation (4.12) lies in the interval $(0,1)$, while the second is greater than 1. Since $k \to \infty$, only the second one is suitable for us, so that

$$P_{loss} = \lim_{k\to\infty} p_k = 1 - \frac{1}{\rho_0} = 1 - \frac{2r}{\lambda\tau + r + 1 + \sqrt{(\lambda\tau + r + 1)^2 - 4\lambda\tau r}}. \tag{4.13}$$

To make the connection with section 3.3, it is amusing to check that the result (4.13) coincides with (3.3), although obtained by quite different arguments!

4.5 The non-symmetrical case

The basic characteristics can be obtained as in the symmetrical case, but formulas are not so explicit. Begin with carrying out cars arrival rates $\{\alpha_i\}$. At steady-state, cars do not accumulated in the nodes, so that we can write flow equations:

$$\alpha_i = \sum_{j=1}^{N} \alpha_j p_{ji}, \ i = \overline{1,N},$$

and $\{\alpha_i\}$ coincide with the stationary distribution $\{\pi_i\}$, up to a multiplicative constant C_α, which can be obtained from the following balance equations satisfied by the cars in movement:

$$\sum_{i,j=1}^{N} \alpha_i p_{ij} \tau_{i,j} = Nr - \sum_{i=1}^{N} L_i, \tag{4.14}$$

where, from (4.6),

$$L_i = \sum_{j=1}^{k} \frac{\rho_i^{k-j}(1-\rho_i)}{1-\rho_i^{k+m+1}} j = \frac{(k+1)(1-\rho_i) - 1 + \rho_i^{k+1}}{(1-\rho_i)(1-\rho_i^{k+m+1})}, \ \rho_i = \frac{\lambda_i}{\alpha_i}, \ i = \overline{1,N}.$$

The left part of equation (4.14) increases from 0 to ∞ as a function of C_α, while its right part decreases from Nr to 0. Thus it has a unique root, which is positive. Now relations (4.4)–(4.9) allow to compute all basic characteristics of the network.

Aknowledgment

The authors would like to thank M. Parent and F. Dumontet (respectively program director and researcher in the PRAXITÈLE project now developed at INRIA) for many fruitful discussions, which contributed to the definition of the above models.

References

[1] A.E. Conway and N.D. Georganas, *Queueing Networks - Exact Computational Algorithms: a Unified Theory Based on Decomposition and Aggregation* (MIT Press, Cambridge, London, 1989).

[2] C. Palm, Intensity variations in telephone traffic, *Ericson Technics* 44 (1943) 1-189 (in German).

[3] W.A. Massey and W. Whitt, Networks of infinite-server queues with nonstationary Poisson input, *Queueing Systems* 13 (1993) 183-250.

[4] F. Kelly, *Reversibility and Stochastic Networks* (John Wiley, 1979).

[5] M Lavrentiev and B. Chabat, *Méthodes de la Théorie des Fonctions d'une Variable Complexe* (Mir Editions, 1977; in French).

Unité de recherche INRIA Rocquencourt
Domaine de Voluceau - Rocquencourt - B.P. 105 - 78153 Le Chesnay Cedex (France)
Unité de recherche INRIA Lorraine - Technopôle de Nancy-Brabois - Campus scientifique
615, rue du Jardin Botanique - B.P. 101 - 54602 Villers lès Nancy Cedex (France)
Unité de recherche INRIA Rennes - IRISA, Campus universitaire de Beaulieu 35042 Rennes Cedex (France)
Unité de recherche INRIA Rhône-Alpes 46, avenue Félix Viallet - 38031 Grenoble Cedex 1 (France)
Unité de recherche INRIA Sophia Antipolis - 2004, route des Lucioles - B.P. 93 - 06902 Sophia Antipolis Cedex (France)

Éditeur
INRIA - Domaine de Voluceau - Rocquencourt - B.P. 105 - 78153 Le Chesnay Cedex (France)

ISSN 0249 - 6399

★ R R . 2 4 6 9 ★